Mathematic Modeling and Performance Analysis of an Adaptive Congestion Control in Intelligent Transportation Systems

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Abstract. In this paper, we develop a preventive congestion control mechanism applied at highway entrances and devised for Intelligent Transportation Systems (ITS). The proposed mechanism provides a vehicular admission control, regulates input traffic and performs vehicular traffic shaping. Our congestion control mechanism includes two classes of vehicles and is based on a specific priority ticket pool scheme with queue-length threshold scheduling policy, tailored to vehicular networks. In an attempt to cope with vehicular traffic fluctuation, we integrate an adaptation ticket generation module. The enhanced mechanism is then able to detect road congestion and provide performance metrics to road site units at the highway entrance. Mathematical modeling based on the embedded Markov chain method highlights the benefits of the proposed scheme. Performance analysis demonstrates the mechanism capability to accurately characterize road traffic congestion conditions, shape vehicular traffic and reduce travel time.

Keywords: Intelligent Transport system, congestion control, performance, mobility model.

1 Introduction

Intelligent Transportation Systems (ITS) address needs of enhancing traffic safety, traffic management and drivers comfort. In order to reach these goals, cooperative vehicular systems promote the exchange of information between vehicles and roadside infrastructure; therefore, Vehicle to Vehicle Communications (V2V) and Vehicles to Infrastructure communications (V2I) constitute two important components of the ITS. An ITS foreseen traffic management application is the vehicular traffic congestion management. In fact, traffic congestion is reflected by transportation costs, travel delay, crash risk, driver stress, and pollution, particularly as a road system approaches its capacity [11]. Therefore, vehicular congestion problem has fostered the design of innovative management strategies. Various point-based techniques are described in the literature such as the use of wireless sensors to control traffic lights [18] and the automated incident detection scheme proposed in [10] to intercept vehicle traffic congestion using the information sent by surveillance cameras. Moreover in [17], authors used the cell dwell time information provided by the telecommunication network to obtain the information associated with vehicle traffic congestion. These types of detectors offer fixed-point or short-section traffic information extracted from vehicles passing the detection zone. Therefore, one of the main limitations of point detection technologies is that traffic estimations are based on measurements related to specific locations; therefore these technologies do not provide an accurate representation of traffic conditions over larger road segments. In this context, cooperative vehicular communication system are of paramount importance where V2V and V2I permit the extension of vehicles data sensors in space and time within wireless vehicular networks [15]. A variety of V2V-based traffic monitoring solutions has been proposed in the literature. In [7], authors suggested that vehicles estimate traffic density extracted

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from the received beacon messages; this valuable information will be continuously exchanged between vehicles. Traffic congestion is detected when the traffic density estimates go beyond road segment average density values. Some techniques adopt the aggregation method that consists of combining data from neighboring vehicles [14][20][13]. In [13], the operation of aggregation and collection of road traffic data is performed by a selected vehicle. Continuous exchange of data on wireless communication channels implies a large overhead and resource consumption. Consequently, some authors oriented their efforts towards alleviating data transmission [3][19][12]. More specifically, StreeSmart [3] consists to restrict sending traffic information to vehicular congestion situations. In [19], authors proposed to apply pattern recognition techniques on beacon messages at each car in order to estimate locally traffic congestion. Authors in [4] brought to the focus a cooperative detection process that helps computing the number of vehicles involved in a traffic jam by adopting a tree-based counting algorithm. Prediction of congestion was the main objective of some papers: In [1], authors reported a method using a non-linear model while Chrobok et al. [2] suggested a technique based on statistical data collected in the past.

The majority of the authors in the research papers focusing on congestion control were concerned by monitoring vehicular traffic, detecting congestion and resorbing the excess of vehicular traffic. Our paper introduces a vehicular admission control at highway entrances and focuses on dynamically adapting highway resources to the road traffic status while shaping the incoming vehicular traffic. The main purpose of our research is the implementation of a preventive congestion control mechanism tailored for ITS. Applied at highway entrances, the congestion control mechanism is based on a ticket pool scheme and two waiting queues. The mechanism aims at controlling the inflow of vehicles entering the highway, reducing traffic congestion and performing traffic shaping.

In order to deal with different classes of vehicles, the system implements the queue length threshold scheduling that applies a service differentiation. Moreover, the ticket pool scheme uses an adaptation module that adapts the ticket generation rate to the vehicular traffic fluctuation in order to control time trip. We performed a mathematical analysis based on an embedded Markov chain and conducted an extensive set of numerical resolutions to examine the performance of the proposed preventive congestion control that strives to shape vehicular traffic in a specific highway scenario.

Performance results highlight the benefits of the proposed congestion control mechanism; it can be successfully used as an admission control at a highway.

This paper is organized as follows. Section 2 defines the vehicular traffic congestion problem. Section 3 describes our proposed preventive congestion control. In section 4, we give a mathematical model of the congestion control based on an embedded Markov chain and we capture the relationship of maximum vehicular burst size, probability of changing exit, mean vehicular queue length and mean vehicular waiting time to the parameters of ticket pool algorithm in closed form equations. Section 5 is devoted for the numerical results and the performance analysis. Final remarks are addressed in Section 6.

2 Vehicular traffic congestion problem

An in-depth understanding of vehicular congestion and vehicular traffic model is necessary to provide the groundwork for minimizing vehicular congestion. In next subsections, we present these models before describing the preventive congestion control.

2.1 Vehicular congestion definition

The three fundamental characteristics of vehicle traffic are flow rate $\lambda$ (vehicles/hour), speed $v$ (km/h) and density $d$ (vehicles/km). The average values of these quantities can be approximately related by the basic traffic stream model [5]:

$$v = \frac{\lambda}{d}$$  \hfill (1)

With few vehicles on the highway, density approaches zero and speeds approach free flow speed. As additional vehicles enter the highway, traffic density and flow rate...
increase until flow reaches a maximum. As additional vehicles continue to enter, the traffic stream density will continue to increase but the flow rate will begin to decrease. When demand exceeds roadway capacity, increasing traffic densities approach a "jam density" limit. At this limit, all vehicles are stopped (i.e., flow rate is zero) with vehicles tightly packed on the highway, leading to the vehicular congestion.

This can been shown in paper [16] that presented an interesting study on various congestion scenarios. Authors shown how the flow rate of the traffic varies with vehicle density. Different flow rates are observed depending on the maximum speed limit of the vehicles. As can be seen in figure 1, two types of vehicles with two maximum speeds exhibit different flow rates.

Capacity refers to the number of vehicles that could be accommodated in a highway. Consider $L_{high}$ as the highway length and $L_{car}$ as the mean car dimension separating car front from the car back. We then derive the maximum number of vehicles, $M$, that circulate on a $l$ lanes-highway as:

$$M \leq \frac{l.L_{high}}{L_{car} + SD}$$

(2)

Where $SD$ is the minimum safety distance between two vehicles.

One relevant congestion indicator is the congestion intensity. Congestion intensity at a particular location is evaluated using level-of-service (LOS) ratings, a grade from A (best) to F (worst), based on the Volume-to-Capacity (V/C) ratio. A V/C less than 0.85 is considered under-capacity, 0.85 to 0.95 is considered near capacity, 0.95 to 1.0 is considered at capacity, and over 1.0 is considered over-capacity.

3 Ticket pool congestion preventive congestion Model

3.1 Highway model description

Fig. 1 illustrates a one lane highway where vehicles circulate towards the intersection area and approach from the highway exit. The wireless infrastructure hosts multiple Road Side Units (RSUs) exchanging messages.

In this study, we consider that each highway entrance is equipped with an RSUei that controls the volume of vehicle traffic entering the road by applying the preventive congestion control explained in the following sub-section. On the other hand, the highway exit hosts an RSUexi that communicates with the vehicle On Board Units (OBUs) and other road side units and exchanges information basically related to performance parameters, specifically the mean waiting time and the traffic density.

3.2 Road Side Units

RSUs located on highway entrances play a critical role since they control the volume of vehicle traffic entering the road by applying our preventive congestion control: Each entrance RSU permits the access to the highway for vehicles having tickets. The ticket generation rate
should be accurately tuned according to highway load. Basically, the RSU should prevent exceeding the maximum number of vehicles that can use the road segment without causing congestion, i.e., the highway capacity. Vehicular congestion control is achieved through interaction between RSU and OBUs and through adopting the ticket pool congestion control mechanism (Fig.3). More specifically,

- RSU continuously monitors vehicular traffic and communicates performance parameters to OBUs in order to advise them about the state of queues and waiting times.

- The ticket rate adaptation module adapts the ticket generation rate according to traffic status as explained in subsection 3.4.

- Ticket generation rate selection is feedback to the ticket pool congestion control module. The latter serves tickets to vehicles according to the algorithm stated in section 3.3 (fig 4).

- OBUs interact with RSUs and choose the exit having the best performance parameters. Once a vehicle acquires a ticket, it circulates on highways according to an enhanced car following mobility model. The latter controls inter-distance and manages intersections.

### 3.3 Proposed ticket pool preventive congestion control algorithm

We have based our mathematical model on queuing model. In fact, in the literature, vehicular mobility models are usually classified as macroscopic, microscopic or mesoscopic[9]. Macroscopic description models vehicular traffic as a continuous flow and treats it according to fluid dynamics. Microscopic description considers each vehicle as a distinct entity, modeling its behavior in a more precise way, but with a high computational cost. Queue models were introduced in the vehicular traffic field by Gawron[6][8]. In this context, each road is modeled as a FIFO queue, and each vehicle as a queue client or packet. Moreover, each road queue is described by its length and a maximum flow, defined by the number of lanes. Since queue models describe the movement of each vehicle in an independent way, they fall into an intermediate category with respect to macroscopic and microscopic descriptions, which can be referred to as mesoscopic. Although queue models do not reproduce some processes such as shockwaves caused by periodic perturbations, queues models have very low computational cost and satisfying performance.

We assume that vehicles are divided into two classes: ordinary traffic and urgent traffic. As one can expect, urgent vehicles such as ambulances and police cars, should receive higher priority and special treatment from the vehicular wireless network. On the other hand, ordinary vehicles designate vehicles with limited priority. The ticket pool congestion control mechanism consists of two waiting queues with finite capacities $K_1$ and $K_2$ that accommodate urgent and ordinary vehicles respectively (fig. 4). Vehicles arriving in a bursty pattern are first queued in the corresponding waiting queue. At highway entrance, each single vehicle should acquire a ticket to be accepted on the highway, and the ticket following an acceptance is removed from the ticket pool with a finite capacity $M$. Tickets are generated at constant intervals every $T$ (units) and stored in the pool. Vehicles in the urgent or ordinary waiting queue can take the highway only if there are tickets in the pool. In order to apply a service differentiation, we implement the Queue Length Threshold (QLT) scheduling policy.
by considering a threshold $L$ in the ordinary buffer. More precisely, the QLT works as follows:

- Whenever there are any tickets in the ticket pool (this implies there are no vehicles waiting), the vehicle arriving, regardless of traffic type, is permitted to circulate.

- If the number of ordinary vehicles is less than or equal to the threshold value $L$ just before the ticket generation epoch, the vehicle (if any) in urgent traffic waiting queue is authorized to circulate.

- If the number of vehicles of ordinary traffic exceeds the threshold value $L$ or if the waiting queue of urgent traffic is empty just before the ticket generation epoch, the vehicle of ordinary traffic can circulate on the highway.

- When a vehicle driver arrives to a full waiting queue, he/she will change his/her itinerary and choose another exit. The vehicle is then considered to be lost from the system.

Given the shaping function provided by the token pool, the maximum burst of traffic will be $M$ and the sustained mean data rate will be $R = 1/T$. In other words, during any time period $\Delta t$, the amount of vehicular traffic on the highway sent cannot exceed $\Delta t/T + M$. Based on this understanding, highway designer should dimension the ticket generation rate and pool size according to the following constraint:

$$R \Delta t + M \leq \frac{L_{high} \cdot L_{car} + SD}{3}$$

### 3.4 Highway ticket rate adaptation through hysteresis

A fixed ticket rate generation will not reflect the fluctuation in a real highway environment. Therefore, rate adaptation to the vehicular traffic dynamics is envisioned: we propose to adapt ticket rate to the vehicular traffic dynamics.

More specifically, the RSU relies on adaptation module in adjusting ticket generation rate $R$ to the traffic situation. In other terms, the monitored highway trip time is used proactively by the congestion control module as a feed-forward input. Basically, a low (resp. high) value of the trip time will lead to a high (resp. low) value of ticket generation rate.

The adaptation module computes the delay averaged over a chosen amount of time. This averaging is necessary because of the high mobility and the vehicular traffic sporadicity in vehicular environments. A major problem with this approach is that the computed delays often fluctuate. The effect is to cause the adaptation module to regularly trigger a new value of ticket rate back and forth.

A better method is to use an hysteresis margin for ticket generation rate decision.

With the hysteresis approach (fig. 5), the monitored delay should be kept between two thresholds: $T_{min}$ and $T_{max}$. When the delay reaches $T_{min}$, the ticket generation rate will be maximum ($R_{max}$). Conversely, when the delay is $T_{max}$ the generation rate will be minimum ($R_{min}$).

An appropriate hysteresis level setting is an important factor in fine-tuning procedures to improve network performance. A hysteresis margin value ($T_{max} - T_{min}$) that is too large yields in reducing fluctuation in ticket...
generation rate. However, it is likely to cause less interaction with vehicular dynamics and degrade the service quality. On the other hand, when the hysteresis margin value is too small, the adaptation module will actively take into consideration vehicular rate fluctuation. But unfortunately, a disadvantage with relatively low hysteresis margin is that it tends to increase occurrence of oscillating ticket generation rate which thereby increases the processor load on the system.

Consequently, hysteresis margin should be tuned carefully by the highway network stakeholders.

4 Mathematical model analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
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<tbody>
<tr>
<td>( n )</td>
<td>Ticket generation instant</td>
</tr>
<tr>
<td>( \lambda_1 )</td>
<td>Urgent arrival process rate</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
<td>Ordinary arrival process rate</td>
</tr>
<tr>
<td>( N_1(n) )</td>
<td>Urgent vehicles number</td>
</tr>
<tr>
<td>( N_2(n) )</td>
<td>Ordinary vehicles number</td>
</tr>
<tr>
<td>( T(n) )</td>
<td>Number of available tickets</td>
</tr>
<tr>
<td>( M )</td>
<td>Size of ticket pool</td>
</tr>
<tr>
<td>( K_1 )</td>
<td>Size of urgent traffic queue</td>
</tr>
<tr>
<td>( K_2 )</td>
<td>Size of ordinary traffic queue</td>
</tr>
<tr>
<td>( L )</td>
<td>Threshold in ordinary queue</td>
</tr>
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Table 1: Mathematical model parameters

4.1 System state distribution

For mathematical modeling clarity, we will refer respectively to the vehicles of urgent traffic and ordinary traffic as type 1 and type 2. We model the proposed scheme using an embedded Markov chain method. The manipulated symbols are listed in table 1. For mathematical tractability, we consider that vehicle inter-arrival time follows an exponential distribution. Kolmogorov-Smirnov test [21] shows that vehicle inter-arrival time based on the empirical data for a real roadway may follow an exponential law. Thus we suppose arrival time follows an exponential distribution.

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We proceed as follows: In a first step, we determine buffer occupancy statistics. Then, we derive performance metrics. It is noteworthy that we consider the system state at discrete time instants, i.e. at ticket generation instants \( 0, T, 2T, \ldots \). At the nth ticket generation instant, we define:

- \( N_1(n) \) and \( N_2(n) \) as respectively type 1 vehicles number and type 2 vehicles number such that \( 0 \leq N_1(n) \leq K_1 \) and \( 0 \leq N_2(n) \leq K_2 \).
- \( T(n) \) as the number of tickets in the ticket pool. In a steady state, a type 1 (or type 2) vehicle is authorized to wait in its corresponding buffer once the ticket pool is empty \( (T(n) = 0) \), leading to \( N_1(n) > 0 \) (or \( N_2(n) > 0) \). Otherwise, the vehicle will be accepted, one per ticket. The ticket pool contains tickets \( (T(n) > 0) \) whenever type 1 and type 2 waiting queues are empty.

Type 1 waiting queue state and pool tickets are related through the following equation:

\[
B_1(n) = N_1(n) + M - T(n) \quad (4)
\]

It is noteworthy that \( B_1(n) \) respects the following inequalities:

- \( 0 \leq B_1(n) \leq M + K_1 \).
- When \( 0 \leq B_1(n) < M, N_1(n) = 0 \) and \( B_1(n) = M - T(n) \).
- When \( M \leq B_1(n) \leq M + K_1, T(n) = 0 \) and \( B_1(n) = N_1(n) + M \).

Moreover, type 1 waiting queue occupancy and the ticket pool occupancy are related according to the following equation:

\[
N_1(n), T(n) = 0 \quad (5)
\]

The ticket pool vehicular congestion control mechanism is modelled by a two dimensional Markov chain process \( P(n) = (B_1(n), N_2(n)) \), \( n \geq 0 \) which presents the finite state space illustrated in fig. 4. As the reader may deduce, states \( (M - i, 0) \) for \( 0 \leq i \leq M \) correspond to a system with empty urgent waiting queue and \( i \) tickets in the ticket pool. Whereas states \( (M + j, l) \) for \( 0 \leq j \leq K_1 - 1 \) and \( 0 \leq l \leq K_2 - 1 \) describe a system with an empty ticket pool, \( j \) waiting urgent vehicles and \( l \) waiting ordinary vehicles.

When \( n \) tends towards infinity, the Markov chain \( P(n) \) is defined by the steady-state probability vector \( X \):

\[
X = (X_{0,0}, X_{1,0}, \ldots, X_{M-1,0}, X_{M,0}, X_{M,1}, \ldots, X_{M,L}, \ldots, X_{M,K_2-1}, X_{M+1,0}, X_{M+1,1}, \ldots, X_{M+1,L},
\]

\]
... \cdot X_{M+1,K_2-1}, \ldots, X_{M+K_1-1,0}, X_{M+K_1-1,1}, \ldots, X_{M+K_1-1,L}, \ldots, X_{M+K_1-1,K_2-1}, X_{M+K_1,L}, \ldots, X_{M+K_1,K_2-1})

The probability vector \( X \) is obtained by solving the equations:

\[
X.A = X
\tag{6}
\]

\[
X.e = 1
\tag{7}
\]

where \( e \) denotes an identity column vector and \( A \) is the transition probability.

At this point, we derive the system state distribution. Therefore, we define the limiting probabilities \( y_k \) and \( y_{k,l} \) as:

\[
y_k = \lim_{t \to \infty} (B_1(t) = k, N_2(t) = 0) \quad 0 \leq k \leq M - 1
\tag{8}
\]

\[
y_{k,l} = \lim_{t \to \infty} (B_1(t) = k, N_2(t) = l) \quad M \leq k \leq M + K_1; 0 \leq l \leq K_2
\tag{9}
\]

where \( N_1(t) = B_1(t) + M - T(t) \) and \( N_2(t) \) the number of type 2 vehicles at time \( t \). Let \( U_m \) be the probability for having \( m \)-arrivals in total during the elapsed ticket generation interval. We can then compute \( U_m \) as:

\[
U_m = \frac{1}{T} \int_0^T \frac{\left(\lambda T\right)^m}{m!} \exp(-\lambda t)dt
\]

\[
= \frac{1}{\lambda T} (1 - \exp(-\lambda T)) \sum_{l=0}^m \frac{\left(\lambda T\right)^l}{l!}
\tag{10}
\]

with \( \lambda = \lambda_1 + \lambda_2 \) the total arrival rate of urgent and ordinary vehicles.

We derive then the system state probability distribution that takes into account the system state at the last ticket generation epoch and the number of arrivals during the elapsed ticket generation interval:

\[
\text{For} \ 0 \leq n \leq M - 1 \ y_n = \sum_{k=0}^n x_k.U_{n-k}
\tag{11}
\]

Consequently, for \( M \leq n_1 \leq M + K_1 - 1, 0 \leq n_2 \leq K_2 - 1 \) the limiting probabilities are then computed as follows:

\[
y_{n_1,n_2} = \sum_{k=0}^{M-1} x_k.U_{n_1-k,n_2} + \sum_{k=M}^{n_1} \sum_{l=0}^{n_2} x_{k,l}.U_{n_1-k,n_2-l}
\tag{12}
\]

\[
y_{n_1,K_2} = \sum_{k=0}^{M-1} x_k.U_{n_1-k,K_2} + \sum_{k=M}^{n_1} \sum_{l=0}^{K_2-1} x_{k,l}.U_{n_1-k,K_2-l}
\tag{13}
\]

\[
y_{M+K_1,n_2} = \sum_{k=0}^{M-1} x_k.U_{M-k,n_2} + \sum_{k=M}^{M+K_1-1} \sum_{l=0}^{n_2} x_{k,l}.U_{M+K_1-k,n_2-l}
\]

\[
+ \sum_{l=L}^{n_2} x_{M+K_1,l}.U_{0,n_2-l,1,n_2>L}
\tag{14}
\]

Finally, we derive the probability \( U_{k,l} \) of \( i \)-arrivals in total during interval \((0, X_i)\) and then \( k \) and \( l \) arrivals.
of type 1 and type 2 vehicles, respectively, during the next ticket generation interval.

\[ U_{j,l} = \sum_{m=j}^{\infty} U_{m,l} \] (15)

\[ U_{j,L} = \sum_{m=j}^{\infty} U_{j,m} \] (16)

\[ U_{j,l}^i = \sum_{m=j}^{\infty} U_{m,l}^i \] (17)

\[ U_{j,L}^i = \sum_{m=j}^{\infty} U_{j,m}^i \] (18)

4.2 Performance Parameters

Now that the buffer occupancy statistics are determined, we focus on deriving performance metrics: mean queue length, mean waiting time, changing exit probability and maximum vehicular burst size of urgent and ordinary vehicular traffic.

- The maximum vehicular burst size refers to the maximum number of vehicles served during a ticket generation interval. It is derived as follows:

\[ N_s = \sum_{i=1}^{M} (M-i)y_{M-i,0} + \sum_{i=0}^{K_2} \sum_{n=0}^{K_1} M_i y_{M+i,n} \] (19)

- The urgent probability of changing exit, \( P_{ex1} \), (or loss probability) represents the probability of an urgent vehicle driver finding a full waiting queue and be forced to change its itinerary. It is considered as lost from the system:

\[ P_{ex1} = \sum_{n=0}^{K_2} y_{M+K_1,n} \] (20)

- The ordinary probability of changing exit, \( P_{ex2} \), (or loss probability) denotes the probability of an ordinary vehicle driver finding a full waiting queue and be forced to change its itinerary:

\[ P_{ex2} = \sum_{n=0}^{M+K_1} y_{n,K_2} \] (21)

- The mean queue length of urgent vehicular traffic corresponds to the average number of vehicles present in urgent buffer:

\[ M_1 = \sum_{i=0}^{K_1} \sum_{n=0}^{K_2} i y_{M+i,n} \] (22)

- The mean queue length of ordinary vehicular traffic denotes the average number of vehicles present in ordinary buffer:

\[ M_2 = \sum_{i=0}^{K_2} \sum_{n=0}^{K_1} i y_{M+n,i} \] (23)

- The mean urgent waiting time corresponds to the mean time observed by a vehicle driver in the urgent queue:

\[ W_1 = \frac{M_1}{\lambda_1(1 - P_{ex1})} \] (24)

- The mean ordinary waiting time is the mean time observed by a vehicle driver in the ordinary queue:

\[ W_2 = \frac{M_2}{\lambda_2(1 - P_{ex2})} \] (25)

5 Performance Analysis

The mathematical model being intractable, we proceeded to the numerical resolution using Matlab and conducted an extensive batch of simulations in order to evaluate performance parameters. Numerical resolutions are achieved for a single lane high-way and with equal mean input rate for urgent and ordinary traffic (i.e. \( \lambda_1 = \lambda_2 = \lambda/2 \)).

The first set of results showed in figure 7 illustrate the impact of different values of threshold \( L \) on the system performance. As \( L \) increases, ordinary traffic gets less priority and urgent traffic experiences higher priority. Thus, ordinary traffic waiting time \( W_2 \) and exit probability \( P_{ex2} \) (resp. urgent traffic waiting time \( W_1 \) and loss probability \( P_{ex1} \)) increase (resp. decrease) as \( L \) increases.

Fig. 8 exhibits the maximum vehicular burst size and the volume-to-capacity ratio. One can see that this number is limited and bounded by \( M \) (the size of ticket pool). This result is very interesting since it shows that the number of admitted vehicles will not exceed the highway capacity. In fact, our model achieves congestion control by input rate regulation and by tightly controlling the vehicle input rate.

Fig. 8 depicts the volume-to-capacity ratio. The system is under capacity (resp. near capacity) for a total average rate less (resp. greater) than 10.25 vehicles per second.

The second set of results illustrated in figure 9 depicts the impact of urgent traffic waiting queue size.
Figure 7: From left to right: Mean waiting time (sec) $K_1=20$, $K_2=20$, $M=20$ - Loss probability $K_1=20$, $K_2=20$, $M=20$.

Figure 8: From left to right: Maximum burst size $K_1=20$, $K_2=20$, $M=20$ - Volume-to-capacity ratio $(K_1, K_2, M) = (20, 20, 20)$. 
One can see that urgent traffic is very sensitive to the change of its buffer size $K_1$. As $K_1$ increases, the urgent traffic waiting delay increases and exit probability decreases.

Ordinary vehicular traffic is less affected by urgent traffic queue size variation at low load. However, as load increases, ordinary traffic experiences more concurrency from higher priority traffic. Thus, its mean waiting time starts to increase. On the contrary, the ordinary traffic exit probability is not affected by $K_1$ variation.

Figure 10 displays performance parameters obtained with different values of ordinary traffic queue size $K_2$. Numerical resolutions were conducted with $K_1$, $L$ and $M$ respectively equal to 20, 10 and 20. The Threshold $L$ equal to 10 has a significant impact on prioritizing ordinary traffic over urgent traffic and the set of values $K_2(20,23,26)$ does not have a major impact on exit probability. This explains the fact that ordinary traffic is not sensitive to the buffer size variation due to $L$ small value. For low loads, mean waiting time is not affected by $K_2$. As the total arrival rate increases, results show a slight increase in delays for both traffic types.

Figure 11 depicts the mean waiting time and the exit probability of each vehicular traffic type, respectively for various values of the ticket pool size. When $M$ increases, urgent and ordinary vehicular traffic are served by a greater number of tickets. Therefore, the exit probability and the mean waiting time for each traffic decrease as the ticket pool size becomes large.

6 Conclusion

Vehicular congestion is a frustrating phenomenon that may happen to a driver. It induces stress, travel delay and pollution. This paper provides a contribution to the development of a preventive congestion control mechanism based on a ticket pool scheme. The mechanism is applied at the highway entrance and serves two types of vehicular traffic: ordinary and urgent. The proposed congestion control mechanism achieves traffic shaping by controlling the number of vehicles served in each period of time. Moreover, ticket generation rate is adapted to vehicular dynamics. We performed a mathematical modelling based on an embedded Markov chain. The performance analysis studies the impact of various system parameters on the system performance. When the ticket pool size increases, exit probability and mean waiting time for each traffic decrease thus improving the system performance. Nevertheless, the pool size depends on the highway capacity and should be well calibrated. On the other hand, we have shown that increasing the ordinary queue threshold induces an increase in ordinary traffic waiting time and exit probability and thus it may induce a starvation problem. As a consequence, the threshold should be tuned according to the highway designer strategy. Finally, the proposed congestion control performance proves that the number of admitted vehicles on a highway does not exceed highway capacity; this illustrates the performance of the algorithm that succeeds to achieve input rate regulation and vehicular traffic shaping.

References

Figure 9: From left to right: Mean waiting time (sec) K2=20, L=18, M=20-Loss probability K2=20, L=18, M=20.

Figure 10: From left to right: Mean waiting time (sec) K1=20, L=10, M=20-Loss probability K1=20, L=10, M=20

Figure 11: From left to right: Mean waiting time (sec) K1=20, K2=20, L=15-Loss probability K1=20, K2=20, L=15


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